| Question |  | Answer | Marks | Guidance |
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| 1 | (i) | A paired sample is used in this context in order to eliminate any effects due to the surfaces used. | E1 <br> [1] | Must refer to (differences between) surfaces. |
| 1 | (ii) | A $t$ test might be used since ... <br> ... the sample is small and <br> ... the population variance is not known (it must be estimated from the data). <br> Must assume: Normality of population ... <br> ... of differences. | $\begin{aligned} & \text { E1 } \\ & \text { E1 } \\ & \text { B1 } \\ & \text { B1 } \\ & \text { [4] } \end{aligned}$ | Allow use of " $\sigma$ ", otherwise insist on "population". <br> Allow "underlying" or "distribution" to imply "population". |
| 1 | (iii) | $\begin{aligned} & \mathrm{H}_{0}: \mu_{D}=0 \\ & \mathrm{H}_{1}: \mu_{D}>0 \end{aligned}$ <br> Where $\mu_{D}$ is the (population) mean reduction/difference in drying time. <br> MUST be PAIRED COMPARISON $t$ test. <br> Differences (reductions) (before - after) are: $\begin{array}{cccccccccc} 0.7 & 0.7 & 0.2 & -0.3 & 0.8 & -0.1 & 0.3 & -0.1 & 0.1 & 0.5 \\ \bar{x}=0.28 & s_{n-1}=0.3852(84) & \left(s_{n-1}{ }^{2}=0.1484(44)\right) \end{array}$ <br> Test statistic is $\frac{0.28-0}{\frac{0.3853}{\sqrt{ } 10}}$ $=2.298$ <br> Refer to $t_{9}$. <br> Single-tailed 5\% point is 1.833 . <br> Significant. <br> Seems mean drying time has fallen. | B1 <br> B1 <br> B1 <br> M1 <br> A1 <br> M1 <br> A1 <br> A1 <br> A1 <br> [9] | Both. Accept alternatives e.g. $\mu_{D}<0$ for $\mathrm{H}_{1}$, or $\mu_{B}-\mu_{A}$ etc provided adequately defined. Hypotheses in words only must include "population". Do NOT allow " $\bar{X}=\ldots$ " or similar. unless $\bar{X}$ is clearly and explicitly stated to be a population mean. For adequate verbal definition. Allow absence of "population" if correct notation $\mu$ is used. <br> Allow "after - before" if consistent with alternatives above. <br> Do not allow $s_{\mathrm{n}}=0.3655\left(s_{n}{ }^{2}=0.1336\right)$ <br> Allow c's $\bar{X}$ and/or $s_{n-1}$. <br> Allow alternative: $0+(\mathrm{c}$ s 1.833) $\times$ <br> $\frac{0.3853}{\sqrt{10}}(=0.2233)$ for subsequent comparison with $\bar{x}$. <br> $\left(\right.$ Or $\bar{x}-(c$ 's 1.833$) \times \frac{0.3853}{\sqrt{10}}$ <br> (= 0.0566) for comparison with 0 .) <br> c.a.o. but ft from here in any case if wrong. Require $3 / 4 \mathrm{sf}$; condone up to <br> 6. Use of $0-\bar{x}$ scores M1A0, but ft . <br> No ft from here if wrong. $\mathrm{P}(t>2.298)=0.02357$. <br> No ft from here if wrong. <br> ft only c's test statistic. <br> ft only c's test statistic. "Non-assertive" conclusion in context to include "on average" oe. |


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| 1 | (iv) |  | $\begin{aligned} & \text { CI is given by } 0.28 \pm \\ & \qquad \begin{array}{l} 2.262 \\ \quad \times \frac{0.3853}{\sqrt{10}} \\ \quad=0.28 \pm 0.2756=(0.0044,0.5556) \end{array} \end{aligned}$ | M1 <br> B1 <br> M1 <br> A1 [4] | Allow c's $\bar{x}$. <br> Allow c's $s_{n-1}$. <br> c.a.o. Must be expressed as an interval. Require $3 / 4 \mathrm{dp}$; condone 5 . If the final answer is centred on a negative sample mean then do not award the final A mark. <br> ZERO/4 if not same distribution as test. <br> Same wrong distribution scores maximum M1 B0 M1 A0. <br> Recovery to $t_{9}$ is OK . |
| 2 | (a) | (i) | For example, need to take a sample because the population might be too large for it to be sensible to take a complete census. <br> Because the sampling process might be destructive. | E1 <br> E1 [2] | Reward 1 mark each for any two distinct, sensible points. |
| 2 | (a) | (ii) | For example <br> Sample should be unbiased. <br> Sample should be representative (of the population). | E1 <br> E1 <br> [2] | Reward 1 mark each for any two distinct, sensible points that the sample/data should be fit for purpose. <br> Further examples include: data should not be distorted by the act of sampling; data should be relevant. |
| 2 | (a) | (iii) | A random sample ... enables proper statistical inference to be undertaken ...... because we know the probability basis on which it has been selected | E2 [2] | Award E2, 1, 0 depending on the quality of response. |
| 2 | (b) | (i) | A Wilcoxon signed rank test might be used when nothing is known about the distribution of the background population. <br> Must assume symmetry (about the median). | E1 <br> E1 <br> [2] | Do not allow "sample", or "data" unless it clearly refers to the population. Do not allow if "Normality" forms part of the assumption. |


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| 2 | (b) | (ii) |  |  |  | $\begin{aligned} & \text { B1 } \\ & \text { B1 } \end{aligned}$ | Both. Accept hypotheses in words. <br> Adequate definition of $m$ to include "population". |
|  |  |  |  |  |  |  |  |
|  |  |  | where $m$ is the population median |  |  |  |  |
|  |  |  | 32.0 | 3.3 | 8 |  |  |
|  |  |  | 29.1 | 0.4 | 3 |  |  |
|  |  |  | 26.1 | -2.6 | 6 |  |  |
|  |  |  | 35.2 | 6.5 | 12 |  |  |
|  |  |  | 34.4 | 5.7 | 11 | M1 | for subtracting 28.7. |
|  |  |  | 28.6 | -0.1 | 1 | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ | for ranks. <br> ft if ranks wrong. <br> If candidate has tied ranks then penalise A0 here but ft from here. |
|  |  |  | 32.3 | 3.6 | 9 |  |  |
|  |  |  | 28.5 | -0.2 | 2 |  |  |
|  |  |  | 27.0 | -1.7 | 5 |  |  |
|  |  |  | 33.3 | 4.6 | 10 |  |  |
|  |  |  | 28.2 | -0.5 | 4 |  |  |
|  |  |  | 31.9 | 3.2 | 7 |  |  |
|  |  |  | $W_{-}=1+2+4+5+6=18$ |  |  | B1 | $\left(W_{+}=3+7+8+9+10+11+12=60\right)$ |
|  |  |  |  |  |  | M1 | No ft from here if wrong. |
|  |  |  | Refer to Wilcoxon single sample tables for $n=12$. Lower $5 \%$ point is 17 (or upper is 61 if 60 used). |  |  | A1 | ie a 1-tail test. No ft from here if wrong. |
|  |  |  | Result is not significant. <br> No evidence to suggest that the median speed has |  |  | A1 | ft only c's test statistic. |
|  |  |  | increased. |  |  | A1 [10] | ft only c's test statistic. "Non-assertive" conclusion in context to include "on average" oe. |
| 3 | (i) |  | $\begin{aligned} & S \sim \mathrm{~N}\left(11.07,2.36^{2}\right) \quad C \sim \mathrm{~N}\left(57.33,8.76^{2}\right) \\ & R \sim \mathrm{~N}\left(24.23,3.75^{2}\right) \end{aligned}$ |  |  |  | When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables, penalise the first occurrence only. |
|  |  |  | $\mathrm{P}(10<S<13)$ |  |  | M1 |  |
|  |  |  | $=\mathrm{P}\left(\frac{10-11.07}{2.36}<Z<\frac{13-11.07}{2.36}\right)$ |  |  |  | For standardising. Award once, here or elsewhere. |
|  |  |  | $=\mathrm{P}(-0.4534<\mathrm{Z}<0.8178)$ |  |  | A1 |  |
|  |  |  | $=0.4679$ |  |  | $\begin{aligned} & \text { A1 } \\ & \text { [3] } \end{aligned}$ | Cao Accept 0.468(0), 0.4681, 0.4682 , but not 0.4683 . |


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| 3 | (ii) | $\begin{aligned} & \text { Want } \mathrm{P}(R>S+10) \text { i.e. } \mathrm{P}(R-S>10) \\ & R-\mathrm{S} \sim \mathrm{~N}(24.23-11.07=13.16, \\ & \left.\quad 3.75^{2}+2.36^{2}=19.6321\right) \\ & \begin{aligned} \mathrm{P}(\text { this }>10) & =\mathrm{P}\left(\mathrm{Z}>\frac{10-13.16}{\sqrt{19.6321}}=-0.7132\right) \\ & =0.7621 \end{aligned} \end{aligned}$ | M1 <br> B1 <br> B1 <br> A1 <br> [4] | Allow $S-R$ provided subsequent work is consistent. Mean. <br> Variance. Accept $s d=\sqrt{ } 19.6321=4.4308 \ldots$ <br> cao |
| 3 | (iii) | $\begin{aligned} & \text { Want } \mathrm{P}(S+R>2 / 3 C) \text { i.e. } \mathrm{P}(S+R-2 / 3 C>0) \\ & S+R-2 / 3 C \sim \mathrm{~N}(11.07+24.23-2 / 3 \times 57.33=-2.92, \\ & \left.2.36^{2}+3.75^{2}+(2 / 3 \times 8.76)^{2}=53.7377\right) \\ & \begin{aligned} \mathrm{P}(\text { this }>0) & =\mathrm{P}\left(Z>\frac{0-(-2.92)}{\sqrt{53.7377}}=0.3983\right) \\ \quad & =1-0.6548=0.3452 \end{aligned} \end{aligned}$ | M1 <br> B1 <br> B1 <br> A1 <br> [4] | Allow $2 / 3 L-(S+R)$ provided subsequent work is consistent. Mean <br> Variance. Accept $s d=\sqrt{ } 53.7377=7.3306 \ldots$ <br> cao |
| 3 | (iv) | $\begin{aligned} & \bar{x}=98.484, s_{n-1}=10.1594 \\ & \text { CI is given by } 98.484 \pm \\ & \quad 2.201 \\ & \quad \times \frac{10.1594}{\sqrt{12}} \\ & \quad=98.484 \pm 6.455=(92.03,104.94) \end{aligned}$ | B1 <br> M1 <br> B1 <br> M1 <br> A1 <br> [5] | Do not allow $s_{n}=9.7269$. <br> ft c 's $\bar{x} \pm$. <br> From $t_{11}$. <br> ft c's $s_{n-1}$. <br> cao Must be expressed as an interval. Require 1 or 2 dp ; condone 3dp. |
| 3 | (v) | Normality is unlikely to be reasonable - times could well be (positively) skewed. <br> Independence is unlikely to be reasonable - e.g. a competitor who is fast in one stage may well be fast in all three. | E1 <br> E1 <br> [2] | Discussion required. Accept any reasonable point. Accept "reasonable" provided an adequate explanation is given. Discussion required. Accept any reasonable point. This is independence between stages for a particular competitor, not between competitors. |



