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June 2012

| Question | | Answer | | Guidance | | | |
|----------|-------|---|-----------------------------------|--|--|--|--|
| 1 | (i) | A paired sample is used in this context in order to eliminate any effects due to the surfaces used. | E1 [1] | Must refer to (differences between) surfaces. | | | |
| 1 | (ii) | A <i>t</i> test might be used since the sample is small and the population variance is not known (it must be estimated from the data). Must assume: Normality of population of <u>differences</u> . | E1 E1 B1 B1 [4] | Allow use of " σ ", otherwise insist on "population". Allow "underlying" or "distribution" to imply "population". | | | |
| 1 | (iii) | $H_0: \mu_D = 0$ $H_1: \mu_D > 0$ | B1 | Both. Accept alternatives e.g. $\mu_D < 0$ for H ₁ , or $\mu_B - \mu_A$ etc provided adequately defined. Hypotheses in words only must include "population". Do NOT allow " $\overline{X} =$ " or similar. unless \overline{X} is clearly and explicitly stated to be a <u>population</u> mean. | | | |
| | | Where μ_D is the (population) mean reduction/difference in drying time. <u>MUST</u> be PAIRED COMPARISON <i>t</i> test. Differences (reductions) (before – after) are: 0.7 0.7 0.2 -0.3 0.8 -0.1 0.3 -0.1 0.1 0.5 | B1 | For adequate verbal definition. Allow absence of "population" if correct notation μ is used. Allow "after – before" if consistent with alternatives above. | | | |
| | | $\overline{x} = 0.28 s_{n-1} = 0.3852(84) (s_{n-1}^{2} = 0.1484(44))$ Test statistic is $\frac{0.28 - 0}{\frac{0.3853}{\sqrt{10}}}$ | B1 M1 | Do not allow $s_n = 0.3655 (s_n^2 = 0.1336)$ Allow c's \overline{x} and/or s_{n-1} . Allow alternative: $0 + (c's 1.833) \times \frac{0.3853}{\sqrt{10}}$ (= 0.2233) for subsequent comparison with \overline{x} . (Or $\overline{x} - (c's 1.833) \times \frac{0.3853}{\sqrt{10}}$ | | | |
| | | = 2.298. Refer to <i>t</i> ₉ . Single-tailed 5% point is 1.833. Significant. Seems mean drying time has fallen. | A1 M1 A1 A1 A1 [9] | (= 0.0566) for comparison with 0.) c.a.o. but ft from here in any case if wrong. Require 3/4 sf; condone up to 6. Use of $0 - \overline{x}$ scores M1A0, but ft. No ft from here if wrong. $P(t > 2.298) = 0.02357$. No ft from here if wrong. ft only c's test statistic. ft only c's test statistic. "Non-assertive" conclusion in context to include "on average" oe. | | | |

| G | Question | | Answer | | Guidance |
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| 1 | (iv) | | CI is given by $0.28 \pm 2.262 \times \frac{0.3853}{\sqrt{10}}$ | M1 B1 M1 | Allow c's \overline{x} . Allow c's s_{n-1} . |
| | | | = 0.28 ± 0.2756 = (0.0044, 0.5556) | A1 | c.a.o. Must be expressed as an interval. Require $3/4$ dp; condone 5. If the final answer is centred on a negative sample mean then do not award the final A mark. ZERO/4 if not same distribution as test. Same wrong distribution scores maximum M1 B0 M1 A0. Recovery to t_9 is OK. |
| 2 | (a) | (i) | For example, need to take a sample because the population might be too large for it to be sensible to take a complete census. Because the sampling process might be destructive. | E1 E1 | Reward 1 mark each for any two distinct, sensible points. |
| | | <i></i> | | [2] | |
| 2 | (a) | (ii) | For example Sample should be unbiased. | E1 | Reward 1 mark each for any two distinct, sensible points that the sample/data should be fit for purpose. |
| | | | Sample should be representative (of the population). | E1 [2] | Further examples include: data should not be distorted by the act of sampling; data should be relevant. |
| 2 | (a) | (iii) | A random sample enables proper statistical inference to be undertaken because we know the probability basis on which it has been selected | E2 [2] | Award E2, 1, 0 depending on the quality of response. |
| 2 | (b) | (i) | A Wilcoxon signed rank test might be used when nothing is known about the distribution of the background population. Must assume symmetry (about the median). | E1 E1 | Do not allow "sample", or "data" unless it clearly refers to the population. |
| | | | | [2] | Do not allow if "Normality" forms part of the assumption. |

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PMT

Mark Scheme

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| 2 | | | B1 | Both. Accept hypotheses in words. | | | |
| | | | where m is the population median | B1 | Adequate definition of <i>m</i> to include "population". | | |
| | | | Speeds –28.7 Rank of diff | | | | |
| | | | 32.0 3.3 8 | | | | |
| | | | 29.1 0.4 3 | | | | |
| | | | 26.1 -2.6 6 | | | | |
| | | | 35.2 6.5 12 | M1 | for subtracting 28.7. | | |
| | | | 34.4 5.7 11 | 101 1 | for subfracting 28.7. | | |
| | | | $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$ | M1 | for ranks. | | |
| | | | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$ | A1 | ft if ranks wrong. | | |
| | | | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$ | | If candidate has tied ranks then penalise A0 here but ft from here. | | |
| | | | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | | | | |
| | | | 28.2 -0.5 4 | | | | |
| | | | 31.9 3.2 7 | | | | |
| | | | $W_{-} = 1 + 2 + 4 + 5 + 6 = 18$ | B1 | $(W_{+} = 3 + 7 + 8 + 9 + 10 + 11 + 12 = 60)$ | | |
| | | | Refer to Wilcoxon single sample tables for $n = 12$. | M1 | No ft from here if wrong. | | |
| | | | Lower 5% point is 17 (or upper is 61 if 60 used). | A1 | ie a 1-tail test. No ft from here if wrong. | | |
| | | | Result is not significant. | A1 | ft only c's test statistic. | | |
| | | | No evidence to suggest that the median speed has increased. | A1 | ft only c's test statistic. "Non-assertive" conclusion in context to include | | |
| | | | increased. | [10] | "on average" oe. | | |
| 3 | | | $S \sim N(11.07, 2.36^2)$ $C \sim N(57.33, 8.76^2)$ | | When a candidate's answers suggest that (s)he appears to have | | |
| | | | $R \sim N(24.23, 3.75^2)$ | | neglected to use the difference columns of the Normal | | |
| | | | P(10 - 0 - 12) | | distribution tables, penalise the first occurrence only. | | |
| | (i) | | P(10 < S < 13) | | | | |
| | | | $= P\left(\frac{10 - 11.07}{2.36} < Z < \frac{13 - 11.07}{2.36}\right)$ | M1 | For standardising. Award once, here or elsewhere. | | |
| | | | | | | | |
| | | | = P(-0.4534 < Z < 0.8178) | A1 | | | |
| | | | = 0.7931 - (1 - 0.6748) | | | | |
| | | | = 0.4679 | A1 | Cao Accept 0.468(0), 0.4681, 0.4682, but not 0.4683. | | |
| | | | | [3] | | | |

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| Question | | Answer | Marks | Guidance |
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| 3 | (ii) | Want $P(R > S + 10)$ i.e. $P(R - S > 10)$ | M1 | Allow $S - R$ provided subsequent work is consistent. |
| | | $R - S \sim N(24.23 - 11.07 = 13.16,$ | B1 | Mean. |
| | | $3.75^2 + 2.36^2 = 19.6321)$ | B1 | Variance. Accept sd = $\sqrt{19.6321} = 4.4308$ |
| | | $P(\text{this} > 10) = P(Z > \frac{10 - 13.16}{\sqrt{19.6321}} = -0.7132)$ | | |
| | | = 0.7621 | A1 | cao |
| | | | [4] | |
| 3 | (iii) | Want $P(S + R > \frac{2}{3}C)$ i.e. $P(S + R - \frac{2}{3}C > 0)$ | M1 | Allow $\frac{2}{3}L - (S + R)$ provided subsequent work is consistent. |
| | | $S + R - \frac{2}{3}C \sim N(11.07 + 24.23 - \frac{2}{3} \times 57.33 = -2.92,$ | B1 | Mean |
| | | $2.36^2 + 3.75^2 + (\frac{2}{3} \times 8.76)^2 = 53.7377)$ | B1 | Variance. Accept sd = $\sqrt{53.7377} = 7.3306$ |
| | | $P(\text{this} > 0) = P(Z > \frac{0 - (-2.92)}{\sqrt{53.7377}} = 0.3983)$ | | |
| | | = 1 - 0.6548 = 0.3452 | A1 | cao |
| | | | [4] | |
| 3 | (iv) | $\overline{x} = 98.484$, $s_{n-1} = 10.1594$ | B1 | Do not allow $s_n = 9.7269$. |
| | | CI is given by $98.484 \pm$ | M1 | ft c's $\overline{x} \pm$. |
| | | 2.201 | B1 | From t_{11} . |
| | | $\times \frac{10.1594}{\sqrt{12}}$ | M1 | ft c's s_{n-1} . |
| | | $\overline{\sqrt{12}}$ | | |
| | | $= 98.484 \pm 6.455 = (92.03, 104.94)$ | A1 | cao Must be expressed as an interval. |
| | | | | Require 1 or 2 dp; condone 3dp. |
| | | | [5] | |
| 3 | (v) | Normality is unlikely to be reasonable – times could | E1 | Discussion required. Accept any reasonable point. |
| | | well be (positively) skewed. | | Accept "reasonable" provided an adequate explanation is given. |
| | | Independence is unlikely to be reasonable $-$ e.g. a | E1 | Discussion required. Accept any reasonable point. |
| | | competitor who is fast in one stage may well be fast | | This is independence between stages for a particular competitor, |
| | | in all three. | [0] | not between competitors. |
| | | | [2] | |

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| Question | | Answer | M | arks | Guidance | | | | | |
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| 4 | (i) | H_0 : The model for the number of callouts fits the da H_1 : The model for the number of callouts does not a the data. | | B1 B1 | Do not allow "Data fit the model" o.e for either hypothesis. | | | the model" o.e for either hypothesis. | | |
| | | Obs'd frequency 145 79 | os'd frequency 145 79 22 | | 5 | 3 | 0 | | | |
| | | | 25.190 | 5.0 |)38 (| 0.756 | 0.101 | | | |
| | | Merge last 3 cells. Obs 9 Exp 5.895 $X^2 = 0.1824 + 0.2939 + 0.4040 + 1.6355$ | | M1 M1 | Calcul | lation o | f X^2 . | | | |
| | | = 2.515(8) | | A1 | Cao R | Require | 3/4 sf; coi | ndone up to 6. | | |
| Refer to χ^2_2 . M1 | | | | | | | | (1s - 2) from wrongly grouped table and ft. ng. P($X^2 > 2.5158$) = 0.2842. | | |
| | | Upper 5% point is 5.991. | | A1 | | | ere if wror | | | |
| | | Not significant. | | A1 | ft only c's test statistic. | | | | | |
| | | Suggests it is reasonable to suppose that the model | | A1 | | | "Non-assertive" conclusion in words (+context). | | | |
| | | fits the data. | | [9] | Do no | model" o.e. | | | | |
| 4 | (ii) | Mean = $5/3$ \therefore $\lambda = 0.6$ | | B1 | | | | | | |
| _ | | | | [1] | G | | 1 . 1 1 | | | |
| 4 | (iii) | $F(t) = \int_0^t 0.6e^{-0.6x} dx$ | | M1 | Correct integral with limits (which may be implied subsequently). Allow use of " $+ c$ " accompanied by a valid attempt to evaluate it. Correctly integrated. | | | | | |
| | | $= \left[-e^{-0.6x} \right]_0^t$ | | A1 | Correc | ctly inte | egrated. | | | |
| | | | | | | | | ated correctly. Accept unsimplified form. in terms of λ then allow max M1A1A0. | | |
| | | | | [3] | In that answer is given in terms of χ then allow max with the | | | | | |
| 4 | (iv) | P(T > 1) = 1 - F(1) | | M1 | ft c's $F(t)$. cao Allow any exact form of the correct answer. | | | | | |
| | | $= 1 - \left(1 - e^{-0.6}\right) = 0.5488$ | | A1 | | | | | | |
| | | · · · · | | [2] | | | | | | |
| 4 | (v) | $F(m) = \frac{1}{2} \qquad \therefore 1 - e^{-0.6m} = \frac{1}{2}$ $\therefore e^{-0.6m} = \frac{1}{2} \qquad \therefore -0.6m = -\ln 2 \qquad \therefore m = \frac{\ln 2}{0.6}$ |] | M1 | | | | | | |
| | | $\therefore e^{-0.6m} = \frac{1}{2} \qquad \therefore -0.6m = -\ln 2 \qquad \therefore m = \frac{\ln 2}{0.6}$ | 1 | M1 | Convi | incing a | ttempt to | rearrange to " $m = \dots$ ", to include use of logs. | | |
| | | m = 1.155 (days) | | A1 | Cao ol Requir | In the correct $F(t)$. Must be evaluated. one 5. | | | | |
| | | | | [3] | <u>^</u> | | | | | |